

# AN OVERVIEW ON BIHARMONIC SUBMANIFOLDS IN SASAKIAN SPACE FORMS

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We present some classification results for biharmonic integral or anti-invariant submanifolds in Sasakian space forms, among which a full classification of proper-biharmonic Legendre curves can be found. We also give explicit examples of such submanifolds.

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# EQUIGEODESICS ON FLAG MANIFOLDS

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## **Abstract**

In this work we give a partial classification of all homogeneous curves on geometric flag manifolds that are geodesics with respect to any invariant metric. We call such geodesics by equigeodesics. This classification of equigeodesics is complete for full flag manifolds. We also characterize the set of all closed equigeodesics.

References: <http://arxiv.org/abs/0904.3770v2>

Gianluca Bande & Amine Hadjar, *On Normal Contact Pairs*:

**Abstract**— We consider manifolds endowed with a contact pair structure. To such a structure are naturally associated two almost complex structures. If they are both integrable, we call the structure a normal contact pair. We generalize the Morimoto’s Theorem on product of almost contact manifolds to flat bundles. We construct some examples on Boothby–Wang fibrations over contact-symplectic manifolds. In particular, these results give new methods to construct complex manifolds.

# GENERALIZED CONSTANT ANGLE SURFACES IN THE EUCLIDEAN 3-SPACE

M. I. MUNTEANU

## Abstract

Starting from the study of surfaces in different product spaces of type  $M^2 \times \mathbb{R}$ , an interesting problem was to explore and classify all constant angle surfaces in the Euclidean 3-space. They are strictly related to general helices and have applications to the theory of liquid crystals. If the Euclidean space is thought as  $\mathbb{R}^2 \times \mathbb{R}$ , a constant angle surface is defined as a surface for which the unit normal makes a constant angle with the  $\mathbb{R}$ -direction. One of the most important properties of these surfaces is the following. Denoting by  $U$  the projection of the fixed direction  $\mathbb{R}$  onto the tangent plane of the surface, it becomes a principal direction with the corresponding principal curvature equal to zero in any point. The aim of the present work is to investigate surfaces isometrically immersed in  $\mathbb{E}^3$  for which the angle function is no longer constant, but  $U$  remains a principal direction everywhere. Finally, classification and characterization results for these surfaces are obtained. This is joint work with A. I. Nistor.

# ON SURFACES IN $\mathbb{H}^2 \times \mathbb{R}$ WITH A CANONICAL PRINCIPAL DIRECTION

A. I. NISTOR

## Abstract

An interesting problem of differential geometry of submanifolds, intensively studied in last years, consists in classification and characterization of constant angle surfaces in different ambient 3-spaces from the Thurston list, such as  $\mathbb{E}^3$ ,  $\mathbb{S}^2 \times \mathbb{R}$ ,  $\mathbb{H}^2 \times \mathbb{R}$  or the Heisenberg group  $Nil_3$ . A constant angle surface is defined as a surface for which the unit normal makes a constant angle with a fixed direction. In case when the ambient is of product type  $M^2 \times \mathbb{R}$ , the  $\mathbb{R}$ -direction is considered. Denoting by  $t$  the global parameter on  $\mathbb{R}$ , it is known that for a constant angle surface the projection  $T$  of  $\frac{\partial}{\partial t}$  onto the tangent space of the surface is a principal direction with the corresponding principal curvature identically zero. The main topic of the present work is to investigate surfaces in  $\mathbb{H}^2 \times \mathbb{R}$  under the assumption that  $T$  remains principal direction in any point, but the corresponding principal curvature is different from zero. We classify these surfaces and we study some properties of them. This is joint work with F. Dillen and M. I. Munteanu.

## **A generalization of a certain Cartan inclusion.**

By C. Durán, T. Püttmann and A. Rigas  
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**Abstract:** A generalized Gromoll - Meyer type action of the unit quaternions on the Euclidean sphere  $S^{4n+2}$ ,  $n \geq 1$ , is free except at one point where the orbit is diffeomorphic to  $S^2$ . The quotient space is included in  $Sp(n)$  homeomorphically and coincides with the Cartan inclusion of the symmetric space  $HP^{n-1} \hookrightarrow Sp(n)$  on the free part of the action. The orbit space inclusion generates  $\pi_{4n+2}Sp(n)$ , which is not stable yet, it can be considered as a 3-suspension of the Cartan inclusion mentioned above, has positive sectional curvature by O'Neill's formula (note that it is not a smooth manifold at exactly one point) and is related to formulas of exotic diffeomorphisms of spheres. All formulas mentioned above are described explicitly and are quite simple.

# Harmonic $f$ -structures

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April 3, 2009

A metric  $f$ -structure is parametrized by a section  $s$  of an associated homogeneous fiber bundle, and conditions for  $s$  to be a harmonic section, and a harmonic map, are studied. These involve the projection onto the subbundle  $\mathcal{G} = \ker f$ , and the almost complex structure in the complementary subbundle  $\mathcal{F} = \operatorname{im} f$ . Metric  $f$ -structures are the generalisation of both almost contact and almost complex metric structures. We will compare the harmonic section equations for both contact and complex cases with those for metric  $f$ -structures. Examples of harmonic metric  $f$ -structures which are neither almost contact nor almost complex metric structures will be given.

# Some Ricci curvature properties of semiconformal maps on 3-dimensional Kenmotsu manifolds

Rodica Voicu

## Abstract

We consider the case of 3-dimensional Kenmotsu manifolds  $M^3(\varphi, \xi, \eta, g)$  which project via a semiconformal map to an orientable surface  $N^2$  endowed with a Hermitian structure  $(J, h)$ .

We will suppose that this map,  $\psi : M^3 \rightarrow N^2$  is a  $(\varphi, J)$ -holomorphic map ([3]). In this case, we characterize the Ricci curvature in terms of Gauss curvature of the orientable surface, the dilation of the semiconformal map, etc (following [1]). Moreover, we obtain some results in the case when  $\psi$  is horizontally homothetic.

Also, we study the Weitzenböck formula on 3-dimensional Kenmotsu manifolds and some examples are discussed.

## References

- [1] P. Baird, L. Danielo, *Three-dimensional Ricci solitons which project to surfaces*, J. Reine Angew. Math. 608 (2007), 65–91.
- [2] P. Baird, J. C. Wood, *Harmonic Morphisms between Riemannian Manifolds*, vol. 29 of London Mathematical Society Monographs, The Clarendon Press, Oxford University Press, Oxford, UK, 2003.
- [3] S. Ianus, A.M.Pastore, *Harmonic maps on contact metric manifolds*, Annales Mathématiques Blaise Pascal, vol. 2, no. 2, pp. 43–53, 1995.
- [4] ] K. Kenmotsu, *A class of almost contact Riemannian manifolds*, Tohoku Math. Journ. 2 , 1972 , 93-103