MATP25 – Linear Functional Analysis, Specialized Course, Fall semester 2017
Programme HT17

Plan for 6./11.

Presentation 1
Theorem 10 and 11 in Section 16.3.3.

Exercise 1
Let \(1 \leq p \leq \infty\). Let \(A = (a_{ij})_{i,j \geq 0}\) be an infinite matrix such that for any \(x = (x_j) \in \ell^p\),
the sequence \(Ax = (Ax)_i\) with
\[
(Ax)_i = \sum_{j=1}^{\infty} a_{ij} x_j
\]
is also in \(\ell^p\). Show that \(A\) defines a bounded linear operator \(\ell^p \to \ell^p\).

Exercise 2
Let \(X\) be a Banach space and let \(E : X \to X\) be a linear map such that \(E^2 = E\), range \(E\) is closed, and the nullspace \(N_E\) is closed. Show that \(E\) is bounded and that range \(E\) is topologically complemented.

Exercise 3
Let \(X, U\) be Banach spaces. We say that a linear map \(M : X \to U\) is bounded below, if there exists a constant \(k > 0\) with \(|Mx| \geq k|x|\) for all \(x \in X\). Show: A bounded linear map \(M : X \to U\) is bounded below, if and only if \(N_M = \{0\}\), and range \(M\) is closed.