LUNDS
UNIVERSITET

Matematik NF

Tentamensskrivning
Topologi
Fredagen den 25 april 2003
Skrivtid: 14.00–19.00

Inga hjälpmedel. Använd institutionens papper, skriv bara på den ena sidan och högst en uppgift på varje papper. Fyll i omslaget fullständigt och skriv initialer på varje ark. Skriv tydligt. Ge klara och kortfattade motiveringar, rita gärna figur i förekommande fall.

1. For $n > 1$ let $\mathbb{R}^n$ be equipped with the standard topology induced by the Euclidean metric and $f : \mathbb{R}^n \to \mathbb{R}$ be the function given by

$$f : (x_1, x_2, \ldots, x_n) \mapsto x_n^2 + \cdots + x_2^2 - x_1^2.$$

Put $A = f^{-1}(\{0\})$, $B = f^{-1}(\{-1\})$ and $C = A \cap \{x \in \mathbb{R}^n | \; x_1^2 = 1\}$. Prove whether the following statements are true or false.

a) A is closed,
b) B is compact,
c) B is connected,
d) C is path-connected,
e) C is compact.

2. Let X, Y be topological spaces and $A_1, A_2, \ldots, A_n$ be closed subsets of X such that $X = \cup_{k=1}^n A_k$. Let $f : X \to Y$ be a function such that the restrictions $f|_{A_k} : A_k \to Y$ of f to A_k are continuous for all $k = 1, 2, \ldots, n$. Prove that $f : X \to Y$ is continuous.

3. Let $(X, \mathcal{T}_X)$, $(Y, \mathcal{T}_Y)$ be topological spaces and $f : X \to Y$ be an open map i.e. it maps open sets to open sets. Let A be a closed subset of X and M be a non-empty subset of Y such that $f^{-1}(M) \subset A$. Prove that there exists a closed subset B of Y such that $M \subset B$ and $f^{-1}(B) \subset A$.

4. Let X be the set of all lines in the plane $\mathbb{R}^2$ which pass through the origin and let $d : X \times X \to [0, \pi/2]$ be the function such that $d(l_1, l_2)$ is the usual Euclidean angle between l_1 and l_2. Prove that (X, d) is a metric space which is homeomorphic to the unit circle $S^1 = \{(x, y) \in \mathbb{R}^2 | \; x^2 + y^2 = 1\}$ in $\mathbb{R}^2$ with the usual topology.

5. A topological space $(X, \mathcal{T})$ is said to be *normal* if for each pair A, B of disjoint closed subsets of X there exist disjoint open subsets U, V of X such that $A \subset U$ and $B \subset V$. Prove that every compact Hausdorff space is normal.